Exercise 57

Find the derivative of the function. Simplify where possible.

$$y = x \sin^{-1} x + \sqrt{1 - x^2}$$

Solution

Use the product rule, the chain rule, and the derivatives of the inverse trigonometric functions listed on page 214.

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left(x \sin^{-1} x + \sqrt{1 - x^2}\right) \\
&= \frac{d}{dx}(x \sin^{-1} x) + \frac{d}{dx}\sqrt{1 - x^2} \\
&= \left[\frac{d}{dx}(x)\right] \sin^{-1} x + x\left[\frac{d}{dx}(\sin^{-1} x)\right] + \frac{d}{dx}\sqrt{1 - x^2} \\
&= (1) \sin^{-1} x + x\left(\frac{1}{\sqrt{1 - x^2}}\right) + \frac{1}{2}(1 - x^2)^{-1/2} \cdot \frac{d}{dx}(1 - x^2) \\
&= \sin^{-1} x + \frac{x}{\sqrt{1 - x^2}} + \frac{1}{2}(1 - x^2)^{-1/2} \cdot (-2x) \\
&= \sin^{-1} x + \frac{x}{\sqrt{1 - x^2}} - \frac{x}{\sqrt{1 - x^2}} \\
&= \sin^{-1} x
\end{aligned}$$